

Modeling with equations

Define a variable / Build an equation / Solve / Check the context

Name: _____ Date: _____

For each problem, define variables and units, write an equation, solve, and check. Give exact answers unless stated otherwise. Diagrams are not to scale.

01 Consecutive integers

The sum of three consecutive even integers is 114. Find the integers.

Variable(s): _____

Equation: _____

Solve / check: _____

02 Theater tickets

A school sold 120 tickets. Adult tickets cost \$12 and student tickets cost \$8. Total ticket revenue was \$1,160. How many of each type were sold?

Variable(s): _____

Equation: _____

Solve / check: _____

03 Ages

A parent is currently four times as old as their child. In 12 years, the parent will be twice as old as the child. Find their current ages.

Variable(s): _____

Equation: _____

Solve / check: _____

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Student practice | Show your model and reasoning. Diagrams are not to scale.

04 Sale price

A backpack is discounted 25%. Sales tax of 8% is then applied to the discounted price. The final price is \$64.80. Find the original price.

Variable(s): _____

Equation: _____

Solve / check: _____

05 Simple interest

A club invests \$8,000 for one year, part at 3% simple interest and the rest at 5% simple interest. The total interest is \$340. How much is invested at each rate?

Variable(s): _____

Equation: _____

Solve / check: _____

06 Fundraiser profit

A club pays a fixed \$180 design fee plus \$7 per shirt. It sells each shirt for \$19. How many shirts must it sell to earn exactly \$420 profit? Assume every shirt ordered is sold.

Variable(s): _____

Equation: _____

Solve / check: _____

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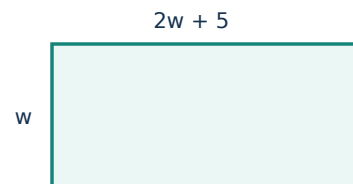
07 Rectangular garden

A rectangular garden has perimeter 70 m. Its length is 5 m more than twice its width. Find its dimensions.

Variable(s): _____

Equation: _____

Solve / check: _____



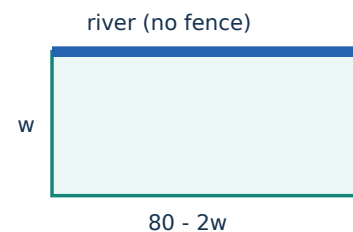
08 Fencing beside a river

A rectangular enclosure uses 80 m of fencing along three sides; the river forms the fourth side, which needs no fence. The area is 600 m². Find all possible dimensions.

Variable(s): _____

Equation: _____

Solve / check: _____



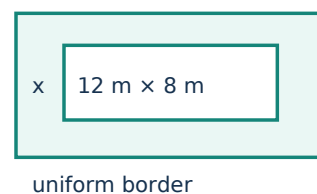
09 A uniform walkway

A rectangular pool is 12 m long and 8 m wide. A walkway of uniform width surrounds the pool. The combined area of pool and walkway is 192 m². Find the walkway width.

Variable(s): _____

Equation: _____

Solve / check: _____



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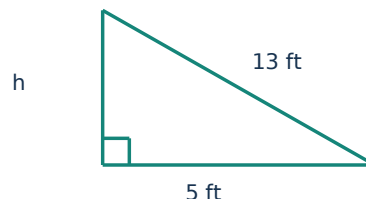
10 Ladder against a wall

A 13-ft ladder rests against a vertical wall on level ground. Its foot is 5 ft from the wall. How high up the wall does it reach?

Variable(s): _____

Equation: _____

Solve / check: _____



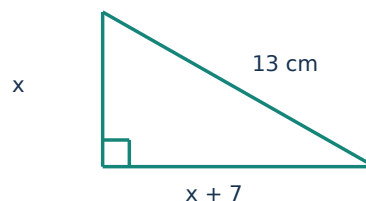
11 Right-triangle banner

A right-triangle banner has one leg 7 cm longer than the other. Its hypotenuse is 13 cm. Find both leg lengths.

Variable(s): _____

Equation: _____

Solve / check: _____



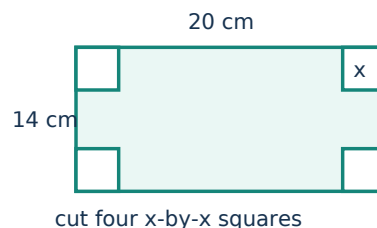
12 Open-top box

A 20-cm by 14-cm sheet is made into an open-top box by cutting equal squares from all four corners and folding up the sides. The base area must be 160 cm^2 . Find the side length of each cutout and the box dimensions.

Variable(s): _____

Equation: _____

Solve / check: _____



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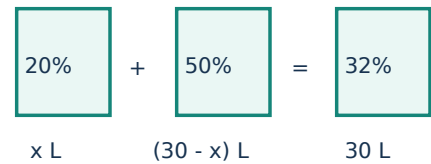
13 Blending solutions

How many liters of 20% salt solution and 50% salt solution should be mixed to make 30 L of 32% salt solution? Assume volumes add and concentrations are measured on the same basis.

Variable(s):

Equation:

Solve / check:



14 Diluting juice

A container holds 12 L of a drink that is 40% fruit juice. How much water must be added to make the drink 30% fruit juice? Assume volumes add.

Variable(s):

Equation:

Solve / check:

15 Evaporating water

A 20-kg saltwater mixture is 15% salt by mass. Only water evaporates. How many kilograms of water must evaporate for the mixture to be 24% salt by mass?

Variable(s):

Equation:

Solve / check:

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Student practice | Show your model and reasoning. Diagrams are not to scale.

16 Trail-mix blend

A store combines nuts costing \$9/kg with dried fruit costing \$5/kg to make 16 kg of a blend costing \$6.50/kg in ingredients. How many kilograms of each are needed?

Variable(s): _____

Equation: _____

Solve / check: _____

17 Replacing liquid

A tank contains 40 L of 25% salt solution. Some well-mixed solution is drained and replaced with the same volume of pure water. The final concentration is 15%. How much solution was drained?

Variable(s): _____

Equation: _____

Solve / check: _____

18 Meeting on a highway

Two cars start at the same time from towns 315 miles apart and travel toward each other at constant speeds of 55 mph and 50 mph. When do they meet, and how far has each traveled?

Variable(s): _____

Equation: _____

Solve / check: _____

55 mph → ← 50 mph

315 miles apart

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Student practice | Show your model and reasoning. Diagrams are not to scale.

19 Catching up

A cyclist leaves a park at 9:00 a.m. riding at 12 mph. Another cyclist leaves the same park along the same route at 9:30 a.m. riding at 18 mph. When and how far from the park does the second cyclist catch the first?

Variable(s): _____

Equation: _____

Solve / check: _____

20 Driving to a tournament

A team drives 180 miles to a tournament and returns along the same route. Its return speed is 15 mph faster than its outbound speed. Total driving time is 7 hours. Find both speeds.

Variable(s): _____

Equation: _____

Solve / check: _____

21 Boat and current

A boat travels 24 miles downstream in 2 hours and 24 miles upstream in 3 hours. Find its speed in still water and the current speed. Assume both are constant.

Variable(s): _____ downstream: $b + c \rightarrow$

Equation: _____ current $\rightarrow$

Solve / check: _____ $\leftarrow$ upstream: $b - c$

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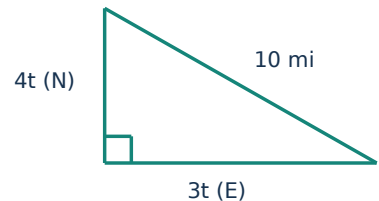
22 Perpendicular walking paths

Two students leave the same point at the same time. One walks east at 3 mph, the other north at 4 mph. After how long are they 10 miles apart?

Variable(s): _____

Equation: _____

Solve / check: _____



23 Two printers

One printer can complete a job in 6 hours; a second can complete the same job in 4 hours. Working together at constant rates, how long will they take?

Variable(s): _____

Equation: _____

Solve / check: _____

24 An unknown pump

Two pumps together fill an empty tank in 3 hours. The first pump alone takes 5 hours. How long does the second pump alone take? Assume constant rates.

Variable(s): _____

Equation: _____

Solve / check: _____

Modeling with equations

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Student practice | Show your model and reasoning. Diagrams are not to scale.

25 Filling with a leak

An inlet fills a tank in 4 hours when the drain is closed. The drain empties a full tank in 12 hours when the inlet is closed. Starting empty, how long does filling take when both are open? Assume constant rates.

Variable(s): _____

Equation: _____

Solve / check: _____

26 Painting partners

One painter takes 3 hours longer than another to paint the same room alone. Together, they finish in 2 hours. Find each painter's solo time, assuming constant rates.

Variable(s): _____

Equation: _____

Solve / check: _____

27 Average test score

A student's first four test scores are 78, 86, 91, and 85. What score on a fifth equally weighted test gives an average of 88?

Variable(s): _____

Equation: _____

Solve / check: _____

Modeling with equations

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Student practice | Show your model and reasoning. Diagrams are not to scale.

28 Sharing a bus cost

A group shares a \$360 bus rental equally. If 3 more students join, the cost per student decreases by \$4. How many students were originally in the group?

Variable(s): _____

Equation: _____

Solve / check: _____

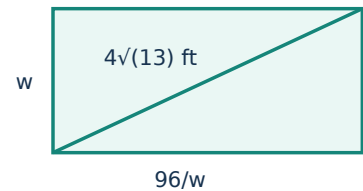
29 A rectangular sign

A rectangular sign has area 96 ft^2 and diagonal length $4\sqrt{13} \text{ ft}$. Its length exceeds its width. Find its dimensions.

Variable(s): _____

Equation: _____

Solve / check: _____



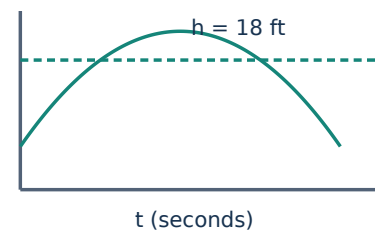
30 A ball launched upward

A ball is launched upward from a 6-ft platform. Its height after t seconds is $h = -16t^2 + 32t + 6$ (feet). At what times is it 18 ft above the ground? State whether it is rising or falling at each time.

Variable(s): _____

Equation: _____

Solve / check: _____



Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

01 Consecutive integers

Define: Let x be the smallest even integer.

Model: $x + (x + 2) + (x + 4) = 114$

Solve: $3x + 6 = 114$, so $3x = 108$ and $x = 36$.

Answer: 36, 38, and 40.

Check: $36 + 38 + 40 = 114$; each successive integer is 2 greater.

02 Theater tickets

Define: Let x be the number of adult tickets; $120 - x$ is the number of student tickets.

Model: $12x + 8(120 - x) = 1160$

Solve: $4x + 960 = 1160$, so $x = 50$. Then $120 - 50 = 70$.

Answer: 50 adult tickets and 70 student tickets.

Check: $50 + 70 = 120$ and $12(50) + 8(70) = 1160$.

03 Ages

Define: Let x be the child's current age in years; the parent is $4x$.

Model: $4x + 12 = 2(x + 12)$

Solve: $4x + 12 = 2x + 24$, so $2x = 12$ and $x = 6$.

Answer: Child: 6 years; parent: 24 years.

Check: In 12 years, their ages are 18 and 36, and $36 = 2(18)$.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

04 Sale price

Define: Let p be the original price in dollars.

Model: $1.08(0.75p) = 64.80$

Solve: $0.81p = 64.80$, so $p = 80$.

Answer: The original price was \$80.

Check: After the discount: \$60. Tax: \$4.80. Final price: \$64.80.

05 Simple interest

Define: Let x be the dollars invested at 3%; $8000 - x$ is invested at 5%.

Model: $0.03x + 0.05(8000 - x) = 340$

Solve: $-0.02x + 400 = 340$, so $x = 3000$.

Answer: \$3,000 at 3% and \$5,000 at 5%.

Check: Interest is $\$90 + \$250 = \$340$; principal totals \$8,000.

06 Fundraiser profit

Define: Let n be the number of shirts sold, a nonnegative integer.

Model: $19n - (180 + 7n) = 420$

Solve: $12n - 180 = 420$, so $12n = 600$ and $n = 50$.

Answer: 50 shirts.

Check: Revenue is \$950; cost is \$530; profit is \$420.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

07 Rectangular garden

Define: Let w be the width in meters; length is $2w + 5$.

Model: $2w + 2(2w + 5) = 70$

Solve: $6w + 10 = 70$, so $w = 10$ and length = 25.

Answer: Width 10 m; length 25 m.

Check: $2(10) + 2(25) = 70$ and $25 = 2(10) + 5$.

08 Fencing beside a river

Define: Let w be a side perpendicular to the river; the fenced length is $80 - 2w$. Require $0 < w < 40$.

Model: $w(80 - 2w) = 600$

Solve: $w^2 - 40w + 300 = 0$; $(w - 10)(w - 30) = 0$. Thus $w = 10$ or 30.

Answer: 10 m by 60 m, or 30 m by 20 m (width by length).

Check: $2(10) + 60 = 80$ and $10(60) = 600$; $2(30) + 20 = 80$ and $30(20) = 600$.

09 A uniform walkway

Define: Let $x > 0$ be the walkway width in meters.

Model: $(12 + 2x)(8 + 2x) = 192$

Solve: $4x^2 + 40x - 96 = 0$; $x^2 + 10x - 24 = 0$; $(x + 12)(x - 2) = 0$. Reject $x = -12$.

Answer: The walkway is 2 m wide.

Check: Outer dimensions are 16 m by 12 m; $16(12) = 192$.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

10 Ladder against a wall

Define: Let $h > 0$ be the height in feet.

Model: $h^2 + 5^2 = 13^2$

Solve: $h^2 = 169 - 25 = 144$; take the positive square root.

Answer: 12 ft.

Check: $12^2 + 5^2 = 144 + 25 = 169 = 13^2$.

11 Right-triangle banner

Define: Let $x > 0$ be the shorter leg in centimeters; the other leg is $x + 7$.

Model: $x^2 + (x + 7)^2 = 13^2$

Solve: $2x^2 + 14x - 120 = 0$; $x^2 + 7x - 60 = 0$; $(x + 12)(x - 5) = 0$. Reject $x = -12$.

Answer: Leg lengths: 5 cm and 12 cm.

Check: $12 - 5 = 7$ and $5^2 + 12^2 = 13^2$.

12 Open-top box

Define: Let x be the cutout side in centimeters; $0 < x < 7$.

Model: $(20 - 2x)(14 - 2x) = 160$

Solve: $4x^2 - 68x + 120 = 0$; $x^2 - 17x + 30 = 0$; $(x - 2)(x - 15) = 0$. Reject $x = 15$ because $x < 7$.

Answer: Cutouts: 2 cm; box: 16 cm by 10 cm by 2 cm high.

Check: Base area is $16(10) = 160 \text{ cm}^2$; both base dimensions are positive.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

13 Blending solutions

Define: Let x be liters of 20% solution; $30 - x$ is liters of 50% solution.

Model: $0.20x + 0.50(30 - x) = 0.32(30)$

Solve: $-0.30x + 15 = 9.6$, so $x = 18$.

Answer: 18 L of 20% solution and 12 L of 50% solution.

Check: Salt amount: $3.6 + 6 = 9.6$; $9.6/30 = 0.32$.

14 Diluting juice

Define: Let $x \geq 0$ be liters of water added.

Model: $0.40(12) = 0.30(12 + x)$

Solve: $4.8 = 3.6 + 0.30x$, so $x = 4$.

Answer: Add 4 L of water.

Check: The final volume is 16 L and $4.8/16 = 0.30$.

15 Evaporating water

Define: Let x be kilograms of water evaporated; $0 \leq x < 20$.

Model: $0.15(20) = 0.24(20 - x)$

Solve: $3 = 4.8 - 0.24x$, so $x = 7.5$.

Answer: 7.5 kg of water must evaporate.

Check: Final mass: 12.5 kg. Salt remains 3 kg, and $3/12.5 = 0.24$.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

16 Trail-mix blend

Define: Let x be kilograms of nuts; $16 - x$ is kilograms of dried fruit.

Model: $9x + 5(16 - x) = 6.50(16)$

Solve: $4x + 80 = 104$, so $x = 6$.

Answer: 6 kg of nuts and 10 kg of dried fruit.

Check: $6(9) + 10(5) = \$104$; $\$104/16 = \6.50 per kg.

17 Replacing liquid

Define: Let x be liters drained, with $0 \leq x \leq 40$.

Model: $0.25(40 - x) = 0.15(40)$

Solve: $10 - 0.25x = 6$, so $x = 16$.

Answer: Drain and replace 16 L.

Check: $24/40 = 60\%$ of the original salt remains. At the restored volume, $0.60(25\%) = 15\%$.

18 Meeting on a highway

Define: Let $t \geq 0$ be hours after departure.

Model: $55t + 50t = 315$

Solve: $105t = 315$, so $t = 3$. Distances are $55(3)$ and $50(3)$.

Answer: They meet after 3 hours; distances are 165 miles and 150 miles.

Check: $165 + 150 = 315$ miles.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

19 Catching up

Define: Let t be hours after 9:30 a.m.; the first cyclist rides for $t + 0.5$ hours.

Model: $18t = 12(t + 0.5)$

Solve: $18t = 12t + 6$, so $t = 1$. Distance = $18(1)$.

Answer: 10:30 a.m., 18 miles from the park.

Check: First cyclist: $12(1.5) = 18$ miles; second cyclist: $18(1) = 18$ miles.

20 Driving to a tournament

Define: Let $v > 0$ be the outbound speed in mph; return speed is $v + 15$.

Model: $180/v + 180/(v + 15) = 7$

Solve: Multiply by $v(v + 15)$: $7v^2 - 255v - 2700 = 0$. Factor: $(v - 45)(7v + 60) = 0$. Reject $v = -60/7$.

Answer: Outbound: 45 mph; return: 60 mph.

Check: $180/45 + 180/60 = 4 + 3 = 7$ hours.

21 Boat and current

Define: Let b be still-water speed and c be current speed, in mph, with $b > c \geq 0$.

Model: $b + c = 24/2 = 12$; $b - c = 24/3 = 8$

Solve: Add the equations: $2b = 20$, so $b = 10$. Then $c = 12 - 10 = 2$.

Answer: Boat: 10 mph in still water; current: 2 mph.

Check: Downstream: $24/(10 + 2) = 2$ h; upstream: $24/(10 - 2) = 3$ h.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

22 Perpendicular walking paths

Define: Let $t \geq 0$ be elapsed time in hours; distances from the start are $3t$ and $4t$.

Model: $(3t)^2 + (4t)^2 = 10^2$

Solve: $25t^2 = 100$, so $t^2 = 4$. Reject $t = -2$.

Answer: 2 hours.

Check: They travel 6 and 8 miles; $\sqrt{6^2 + 8^2} = 10$ miles.

23 Two printers

Define: Let $t > 0$ be hours working together. Each printer contributes rate times time.

Model: $t/6 + t/4 = 1$

Solve: Multiply by 12: $2t + 3t = 12$, so $t = 12/5 = 2.4$.

Answer: 2.4 hours, or 2 hours 24 minutes.

Check: $2.4/6 + 2.4/4 = 0.4 + 0.6 = 1$ complete job.

24 An unknown pump

Define: Let $x > 0$ be the second pump's solo filling time in hours.

Model: $1/5 + 1/x = 1/3$

Solve: $1/x = 1/3 - 1/5 = 2/15$, so $x = 15/2$.

Answer: 7.5 hours, or 7 hours 30 minutes.

Check: In 3 hours, the pumps fill $3/5 + 3/7.5 = 0.6 + 0.4 = 1$ tank.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

25 Filling with a leak

Define: Let $t > 0$ be the filling time in hours.

Model: $t/4 - t/12 = 1$

Solve: Multiply by 12: $3t - t = 12$, so $2t = 12$ and $t = 6$.

Answer: 6 hours.

Check: Net rate = $1/4 - 1/12 = 1/6$ tank per hour. In 6 hours, one tank fills.

26 Painting partners

Define: Let $x > 0$ be the faster painter's solo time; the slower time is $x + 3$.

Model: $1/x + 1/(x + 3) = 1/2$

Solve: $2(x + 3) + 2x = x(x + 3)$, so $x^2 - x - 6 = 0$. Factor: $(x - 3)(x + 2) = 0$. Reject $x = -2$.

Answer: Faster painter: 3 hours; slower painter: 6 hours.

Check: $1/3 + 1/6 = 1/2$ room per hour, and $6 - 3 = 3$ hours.

27 Average test score

Define: Let x be the fifth test score, with $0 \leq x \leq 100$.

Model: $(78 + 86 + 91 + 85 + x)/5 = 88$

Solve: $340 + x = 440$, so $x = 100$.

Answer: 100 points.

Check: $(78 + 86 + 91 + 85 + 100)/5 = 440/5 = 88$.

Worked solutions

Define a variable / Build an equation / Solve / Check the context

Check the setup as well as the answer. Reject values that violate the situation.

28 Sharing a bus cost

Define: Let $n > 0$ be the original number of students, an integer.

Model: $360/n - 360/(n + 3) = 4$

Solve: Multiply by $n(n + 3)$: $1080 = 4n(n + 3)$. Then $n^2 + 3n - 270 = 0$; $(n + 18)(n - 15) = 0$. Reject $n = -18$.

Answer: 15 students originally.

Check: Original share: $\$360/15 = \24 . New share: $\$360/18 = \20 . Difference: $\$4$.

29 A rectangular sign

Define: Let $w > 0$ be the width in feet; length is $96/w$, and length $>$ width.

Model: $w^2 + (96/w)^2 = (4\sqrt{13})^2 = 208$

Solve: Multiply by w^2 : $w^4 - 208w^2 + 9216 = 0$. Let $u = w^2$: $(u - 64)(u - 144) = 0$. The side lengths are 8 and 12; width is the shorter side.

Answer: Width 8 ft; length 12 ft.

Check: $8(12) = 96$ and $8^2 + 12^2 = 208 = (4\sqrt{13})^2$.

30 A ball launched upward

Define: Let $t \geq 0$ be seconds after launch. Set the given height model equal to 18.

Model: $-16t^2 + 32t + 6 = 18$

Solve: $16t^2 - 32t + 12 = 0$; $4t^2 - 8t + 3 = 0$; $(2t - 1)(2t - 3) = 0$. Thus $t = 0.5$ or 1.5 . The vertex occurs at $t = 1$.

Answer: 0.5 seconds while rising; 1.5 seconds while falling.

Check: $h(0.5) = -4 + 16 + 6 = 18$; $h(1.5) = -36 + 48 + 6 = 18$.